



QCD and Transverse-Spin Physics

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A pedagogical presentation of single-spin asymmetries and transversity is offered. Detailed discussion is given of various aspects of single-spin asymmetries in lepton–nucleon and in hadron–hadron scattering and of the rôle of perturbative QCD and evolution in the context of transversity.

1 Preamble

Let me begin by remarking that many of the topics touched on here are covered in much greater depth in [1, 2]. Therefore, much credit and thanks are due to my two collaborators Barone and Drago. Further, less condensed and more complete reviews than the present may also be found in other recent proceedings: see, *e.g.*, [3].

By way of motivation for the subject area, I should open by noting that the general theoretical framework for discussing transverse-spin effects, at least at a basic level, is now rather solid. Added to which, on the experimental side there is presently a great deal of activity: witness the programmes of HERMES at DESY, COMPASS at CERN and the spin programme at RHIC. It is, however, also true that much theoretical work is still necessary to unravel the phenomenology; both to perform serious and relevant future data analysis and to indicate which measurements could be most usefully performed.

What is *transverse* spin? By “transverse” one means that the spin vector is perpendicular to the particle momentum (*cf.* parallel or longitudinal, as in the talk by Ridolfi [4]). This terminology should not be confused with the traditional designation “transverse state”, as applied to the case of gauge bosons (where the EM fields lie in the transverse plane): left- and right-handed circular polarisations correspond to helicity states and therefore to a *longitudinal* spin vector. It is also important to stress that transverse polarisation itself does *not* depend on particle masses (*cf.* the natural polarisation of the LEP beam). However, the problem of mass can and does arise when seeking *measurable* transverse-spin effects, which almost inevitably require spin flip.

1.1 Transversity

Transversity, which simply describes the probability of finding a quark polarised parallel (as opposed to antiparallel) to a *transversely*-polarised parent hadron, has a rather

long history: the concept (though *not* the term) was introduced in [5] via Drell–Yan processes; its leading-order (LO) anomalous dimensions were first calculated in [6] but, unfortunately, languished *forgotten* for over a decade! They were recalculated much later in [7] and, still early on, *unwittingly* (as part of the evolution of g_2), by a number of authors [8–11].

1.2 The DIS structure function g_2

Dubbed “*the nucleon’s other spin dependent structure function*” by Jaffe [12], the DIS structure function g_2 has an even longer history. Already in 1972 its scaling behaviour had been examined [13–15]; as already noted, the LO evolution in QCD was calculated by various authors [8–11] (although incorrectly in the earlier papers).

Now, It is important to appreciate here that g_2 is *very* different to the better-known F_2 and g_1 DIS structure functions: it is essentially twist-three and therefore involves three-parton correlators; it can thus have *no* partonic interpretation. While it is true that in the Wandzura–Wilczek approximation g_2 may be related to g_1 [16], this is only via explicit neglect of the higher-twist contributions and it is now largely accepted that there is no compelling reason for so doing.

1.3 Single-spin asymmetries

Single-spin asymmetries (SSA’s) perhaps represent the *oldest* form of high-energy spin measurement: the only requirement is either a polarised beam or target (and for Λ^0 production *neither* is necessary). However, after early interest (due to the surprisingly large magnitudes found experimentally), a theoretical *dark age* descended on SSA’s: apparently perturbative QCD (pQCD) had nothing to say, save that they ought to *vanish*. We now realise that the rich phenomenology is matched by a richness of the theoretical framework. This will, in essence, be a central theme of the present talk.

Before continuing I have to admit that one might argue that the Q^2 of existing SSA data is too low for pQCD to be applicable. Indeed, there are many *non*-pQCD models that explain part (but never all) of the data; Some examples may be found in [17–20]. Here, however, I shall examine SSA's purely within the pQCD framework. It is also true that the present data show *no* indication that such effects are dying off with growing Q^2 .

2 Introduction

2.1 Single-spin asymmetries

Generically, SSA's reflect correlations of the form

$$\vec{s} \cdot (\vec{p} \wedge \vec{k}), \quad (1)$$

where \vec{s} is a spin vector, \vec{p} and \vec{k} are particle/jet momenta. Indeed, it should not be difficult to convince oneself that the constraint of parity conservation imposes such a form when only one spin vector is available. A typical example might be: \vec{s} a target polarisation vector (transverse), \vec{p} the beam direction and \vec{k} a final-state particle direction. Therefore, polarisations involved in SSA's must typically be transverse with respect to the reaction plane, although there are exceptions.

Transforming basis from transverse spin to helicity via

$$|\uparrow / \downarrow\rangle = \frac{1}{\sqrt{2}} [|+\rangle \pm i |-\rangle], \quad (2)$$

any such asymmetry takes on the (schematic) form

$$\mathcal{A}_N \sim \frac{\langle \uparrow \uparrow \uparrow \rangle - \langle \downarrow \downarrow \downarrow \rangle}{\langle \uparrow \uparrow \uparrow \rangle + \langle \downarrow \downarrow \downarrow \rangle} \sim \frac{2 \operatorname{Im} \langle + | - \rangle}{\langle + | + \rangle + \langle - | - \rangle}. \quad (3)$$

The form of the second numerator indicates *interference* between amplitudes, where one is *spin-flip* and the other *non-flip*, with a relative phase difference.

It was soon realised [21] that a gauge theory such as QCD in the Born approximation and massless (or high-energy) limit cannot satisfy either requirement: fermion helicity is conserved and tree diagrams are real. This provoked the statement [21] that “... *observation of significant polarizations in [pion production] would contradict either QCD or its applicability.*” Clearly, however, QCD is still alive and well, despite a large number of sizable, measured single transverse-spin effects.

It was not long, however, before Efremov and Teryaev [22] opened up an escape route via consideration of the three-parton correlators involved in, *e.g.*, g_2 : they demonstrated that the relevant mass scale for helicity flip is not the current quark mass, but a typical hadronic mass and that the pseudo-two-loop nature of the diagrams can lead to an imaginary part in certain regions of partonic phase space. Unfortunately, quite some time passed before the richness of the available structures was recognised and brought fully to fruition, see [23].

2.2 Transversity

Transversity is the third (and *final*) twist-two partonic distribution function. At this point it is important to make the distinction between partonic distributions (or densities) (*e.g.*, $q(x)$, $\Delta q(x)$, $\Delta_T q(x)$, ...) and DIS structure functions (F_1 , F_2 , g_1 , g_2 , ...). In the unpolarised and helicity-dependent cases at leading twist there is a simple, rather direct, correspondence between the two: DIS structure functions are just weighted sums of parton densities. However, as already noted, in the case of transverse-spin: (i) there is no DIS transversity structure function and (ii) g_2 cannot be expressed in terms of a partonic densities.

The three twist-two structures are then

$$q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle PS | \bar{\psi}(0) \gamma^+ \psi(0, \xi^-, \mathbf{0}_\perp) | PS \rangle, \quad (4a)$$

$$\Delta q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle PS | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(0, \xi^-, \mathbf{0}_\perp) | PS \rangle, \quad (4b)$$

$$\Delta_T q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle PS | \bar{\psi}(0) \gamma^+ \gamma^1 \gamma_5 \psi(0, \xi^-, \mathbf{0}_\perp) | PS \rangle. \quad (4c)$$

The presence of the γ_5 matrix signals generic spin dependence while the γ^1 in $\Delta_T q(x)$ signals helicity flip, *precluding* transversity contributions in DIS, see Fig. 1. N.B. chirality flip is not a problem if the quarks connect to different hadrons, as in the Drell–Yan (DY) process.

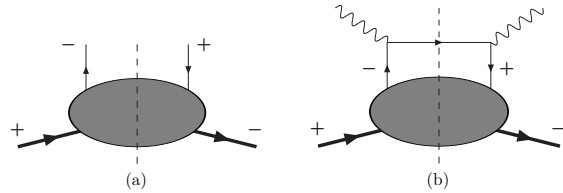


Figure 1. (a) The chirally-odd hadron–quark amplitude for h_1 and (b) the *forbidden* chirality-flip DIS handbag diagram.

2.3 Perturbative QCD evolution

The non-diagonal nature of transversity in a helicity basis forces diagonality in flavour space, see Fig. 2, and thus the LO QCD evolution of transversity is of the non-singlet type. The quark line cannot return to the same hadronic blob and therefore there can be neither quark–gluon mixing nor mixing between different types of quark.

The LO non-singlet DGLAP quark–quark splitting func-

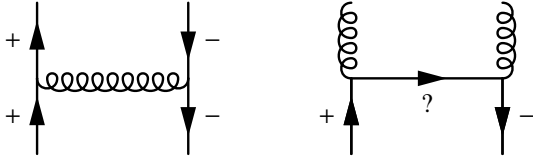


Figure 2. Left, the evolution kernel in a physical (axial) gauge for transversity; right, an excluded gluon–fermion mixing diagram.

tions are:

$$P_{qq}^{(0)} = C_F \left(\frac{1+x^2}{1-x} \right)_+, \quad (5a)$$

$$\Delta P_{qq}^{(0)} = P_{qq}^{(0)} \quad (\text{helicity conservation}), \quad (5b)$$

$$\Delta_T P_{qq}^{(0)} = C_F \left[\left(\frac{1+x^2}{1-x} \right)_+ - 1 + x \right]. \quad (5c)$$

One sees that while the first moments of $P_{qq}^{(0)}$ and $\Delta P_{qq}^{(0)}$ both *vanish* (leading to well-known conservation laws and sum rules), the same does *not* hold for $\Delta_T P_{qq}^{(0)}$. The overall effect is a decrease in transversity with respect to helicity densities as Q^2 increases, see Fig. 3.

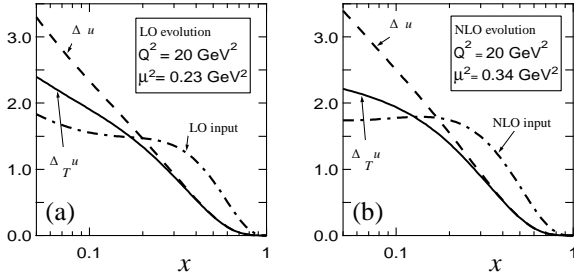


Figure 3. A comparison of the Q^2 -evolution of $\Delta_T u(x, Q^2)$ and $\Delta u(x, Q^2)$ at (a) LO and (b) NLO; from [24].

2.4 The Soffer bound

Soffer [25] has derived an interesting and non-trivial bound involving all three leading-twist structures. In terms of hadron–quark helicity amplitudes, see Fig. 4, the quark

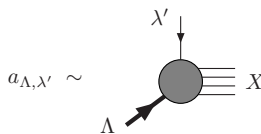


Figure 4. Hadron–parton helicity amplitudes, see [25].

densities may be expressed as

$$q(x) \propto \text{Im}(\mathcal{A}_{++,+} + \mathcal{A}_{+,-,-}) \propto \sum_X (a_{++}^* a_{++} + a_{+-}^* a_{+-}), \quad (6a)$$

$$\Delta q(x) \propto \text{Im}(\mathcal{A}_{++,+} - \mathcal{A}_{+,-,-}) \propto \sum_X (a_{++}^* a_{++} - a_{+-}^* a_{+-}), \quad (6b)$$

$$\Delta_T q(x) \propto \text{Im} \mathcal{A}_{+,-,-} \propto \sum_X a_{--}^* a_{++}. \quad (6c)$$

The following Schwartz identity (combined with parity conservation)

$$\sum_X |a_{++} \pm a_{--}|^2 \geq 0 \Rightarrow \sum_X a_{++}^* a_{++} \pm \sum_X a_{--}^* a_{++} \geq 0, \quad (7)$$

then leads to

$$q_+(x) \geq |\Delta_T q(x)| \quad \text{or} \quad q(x) + \Delta q(x) \geq 2|\Delta_T q(x)|. \quad (8)$$

Note that, while saturation of the bound is, of course, not necessarily expected *a priori*, it is rather suggestive that the physical magnitude of $\Delta_T q(x)$ might well be intermediate to $q(x)$ and $\Delta q(x)$. Indeed, there are many arguments for expecting that $\Delta_T q(x)$ should be of a similar strength to $\Delta q(x)$ (for example, see [1]), at least at some sufficiently low energy scale.

2.5 A DIS definition for transversity

Quark density functions find their natural definition in the lepton–nucleon DIS process, where the parton model is usually formulated and non-perturbative models developed. On translation to DY, it is well known that large K factors of $O(\pi\alpha_s)$ appear. At RHIC energies this represents an approximately 30% correction while at EMC/SMC energies it is nearly 100%. As is well known, such corrections are indeed corroborated by the data.

The pure DY coefficient functions are known for transversity, see [26–29], but are scheme dependent. Moreover, a term $\ln^2 x/(1-x)$ appears, which is *not* found for spin-averaged [30] or helicity-dependent [31] DY. Added to problems arising with a vector–scalar current product [32], This suggests that an interesting check is in order. In order to have a DIS-like process as a starting point, it is clearly necessary to allow for helicity flip somewhere. This may be most conveniently achieved via the introduction of a scalar vertex, see Fig. 5. A certain amount of care is needed as an extra contribution from the scalar vertex must be absorbed into the running mass (or Higgs-like coupling constant).

Armed with such a process, in the standard manner one may now calculate a coefficient function for transversity in DIS, which combined with the already-known coefficient for DY will provide the corresponding K factor. The three

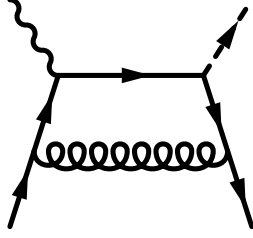


Figure 5. A DIS Higgs–photon interference diagram.

cases of unpolarised, longitudinally and transversely polarised are displayed in the following equations:

$$C_{q,DY}^f - 2C_{q,DIS}^f = \frac{\alpha_s}{2\pi} C_F \left[\left(\frac{4}{3}\pi^2 + 1 \right) \delta(1-x) + \frac{3}{(1-x)_+} + 2(1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ - 6 - 4x \right], \quad (9a)$$

$$C_{q,DY}^g - 2C_{q,DIS}^g = C_{q,DY}^f - 2C_{q,DIS}^f + \frac{\alpha_s}{2\pi} C_F [2 + 2x], \quad (9b)$$

$$C_{q,DY}^h - 2C_{q,DIS}^h = \frac{\alpha_s}{2\pi} C_F \left[\left(\frac{4}{3}\pi^2 - 1 \right) \delta(1-x) + \frac{3x}{(1-x)_+} - 6x \frac{\ln^2 x}{1-x} + 4 - 4x + 4x \left(\frac{\ln(1-x)}{1-x} \right)_+ \right], \quad (9c)$$

where $C_F = \frac{4}{3}$ is the just usual colour-group Casimir for the fermion representation. The small difference in the coefficient of the δ -function is not actually significant, the most striking difference is the appearance, already mentioned, of the $\ln^2 x/(1-x)$ term. We note that while one might object that any substantial differences are probably due to the peculiar DIS definition adopted for transversity, this term arises in the DY calculation and has its origin in the particular phase-space integration required by the fixing of the final lepton-pair azimuthal angle.

2.6 DIS–DY transversity asymmetry

Using the above results it is now possible to evaluate the effect of such a K factor on the DY transversity asymmetry. In Fig. 6 we display the asymmetry A_{DY} for transversely polarised protons in the DY process. The full curve shows the LO case while the dotted line shows the effect of the δ -function contribution and the dot–dashed line represents the full calculation. One can clearly see that, in contrast to the helicity case [31], the terms beyond the δ -function alter the behaviour quite considerably. This is a signal that direct comparison with model calculations might not be as straightforward as hoped.

One might, of course, argue that this is just an artifact of the peculiar DIS definition used. However, the substantial departures from the familiar behaviour occur in the DY

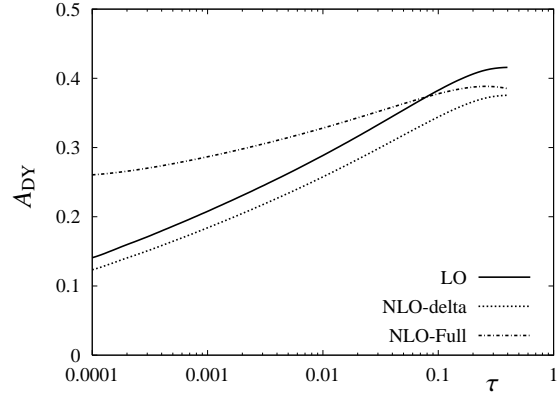


Figure 6. The transversity asymmetry (*valence quarks only*) for Drell–Yan. The variables are $\tau = Q^2/s$, $s = 4 \cdot 10^4 \text{ GeV}^2$, with kinematic limits $\tau < x_1, x_2 < 1$; see the text for a description of the different curves.

calculation (and are traceable to the phase space alterations due to the requirement of *not* integrating over the azimuthal angle of the lepton pair). In any case, work is under way to perform similar calculations for the various possible combinations of scalar and vector currents in DIS and DY, in order to confirm the origins of the large K factor found.

3 Single-Hadron Production

While the cleanest and most unambiguous experimental access to transversity should nevertheless lie in the DY process, SSA's represent a more immediately available (if not necessarily accessible) source of information. Thus, I shall now briefly examine single-hadron production off a transversely polarised target:

$$A^\uparrow(P_A) + B(P_B) \rightarrow h(P_h) + X. \quad (10)$$

Hadron A is transversely polarised and the unpolarised (or spinless) hadron h (which may also be a photon) is produced at *large* transverse momentum P_{hT} , thus pQCD is applicable. The process is shown pictorially in Fig. 7.

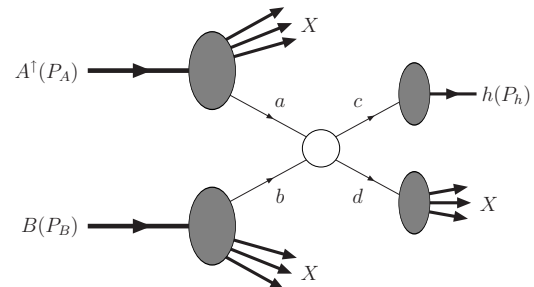


Figure 7. Hadron–hadron scattering with a single polarised initial-state hadron (A^\uparrow).

In a typical experimental example A and B are protons while h is a pion. One measures an SSA:

$$A_T^h = \frac{d\sigma(S_T) - d\sigma(-S_T)}{d\sigma(S_T) + d\sigma(-S_T)}. \quad (11)$$

According to the factorisation theorem, the differential cross-section for the reaction may be written formally as

$$d\sigma = \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \rho_{\alpha\alpha'}^a f_a(x_a) \otimes f_b(x_b) \otimes d\hat{\sigma}_{\alpha\alpha'\gamma\gamma'} \otimes \mathcal{D}_{h/c}^{\gamma\gamma'}(z). \quad (12)$$

Here f_a (f_b) is the density of parton a (b) in hadron A (B), $\rho_{\alpha\alpha'}^a$ is the spin density matrix of parton a , $\mathcal{D}_{h/c}^{\gamma\gamma'}$ is the fragmentation matrix of parton c into hadron h and $d\hat{\sigma}/d\hat{t}$ is the elementary cross-section:

$$\left(\frac{d\hat{\sigma}}{d\hat{t}}\right)_{\alpha\alpha'\gamma\gamma'} = \frac{1}{16\pi s^2} \frac{1}{2} \sum_{\beta\delta} \mathcal{M}_{\alpha\beta\gamma\delta} \mathcal{M}_{\alpha'\beta'\gamma'\delta}^*, \quad (13)$$

where $\mathcal{M}_{\alpha\beta\gamma\delta}$ is the amplitude for the hard partonic process, shown in Fig. 8.

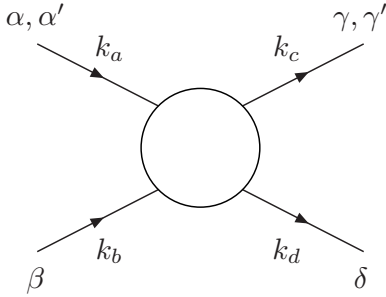


Figure 8. The hard-scattering parton amplitude $\mathcal{M}_{\alpha\beta\gamma\delta}$.

For an unpolarised produced hadron, the off-diagonal elements of $\mathcal{D}_{h/c}^{\gamma\gamma'}$ vanish, *i.e.*, $\mathcal{D}_{h/c}^{\gamma\gamma'} \propto \delta_{\gamma\gamma'}$. Then, helicity conservation implies $\alpha = \alpha'$ and there can be no dependence on the spin of hadron A and thus all SSA's are identically zero. Such a conclusion, in stark contrast with reality, may be avoided by considering either intrinsic quark transverse motion, or higher-twist effects.

3.1 Transverse motion and SSA's

Quark intrinsic transverse motion can generate SSA's in three different ways:

1. κ_T in hadron h implies $\mathcal{D}_{h/c}^{\gamma\gamma'}$ may be non-diagonal (T -odd effect at the fragmentation level).
2. \mathbf{k}_T in hadron A requires $f_a(x_a)$ to be replaced by $\mathcal{P}_a(x_a, \mathbf{k}_T)$, which may depend on the spin of A (T -odd effect at the distribution level).
3. \mathbf{k}'_T in hadron B requires $f_b(x_b)$ to be replaced by $\mathcal{P}_b(x_b, \mathbf{k}'_T)$. The transverse spin of b in the unpolarised B may then couple to the transverse spin of a (T -odd effect at the distribution level).

The three mechanisms are, correspondingly:

1. the Collins effect [33];
2. the Sivers effect [34];
3. an effect in Drell–Yan studied by Boer [35].

Note that all such intrinsic- κ_T , $-\mathbf{k}_T$, or $-\mathbf{k}'_T$ effects are T -odd and therefore they require initial- or final-state interactions. When quark transverse motion is included, the QCD factorisation theorem is *not* proven.

Assuming factorisation to be valid, the cross-section is

$$E_h \frac{d^3\sigma}{d^3\mathbf{P}_h} = \sum_{abc} \sum_{\alpha\alpha'\beta\beta'\gamma\gamma'} \frac{1}{\pi z} \times \int dx_a \int dx_b \int d^2\mathbf{k}_T \int d^2\mathbf{k}'_T \int d^2\mathbf{\kappa}_T \times \mathcal{P}_a(x_a, \mathbf{k}_T) \rho_{\alpha\alpha'}^a \mathcal{P}_b(x_b, \mathbf{k}'_T) \rho_{\beta\beta'}^b \times \left(\frac{d\hat{\sigma}}{d\hat{t}}\right)_{\alpha\alpha'\beta\beta'\gamma\gamma'} \mathcal{D}_{h/c}^{\gamma\gamma'}(z, \mathbf{\kappa}_T), \quad (14)$$

where

$$\left(\frac{d\hat{\sigma}}{d\hat{t}}\right)_{\alpha\alpha'\beta\beta'\gamma\gamma'} = \frac{1}{16\pi s^2} \sum_{\beta\delta} \mathcal{M}_{\alpha\beta\gamma\delta} \mathcal{M}_{\alpha'\beta'\gamma'\delta}^*. \quad (15)$$

The Collins mechanism requires that we take into account the intrinsic quark transverse motion inside the produced hadron h , and neglect the transverse momenta of all other quarks (assuming the spin of A to be directed along y):

$$E_h \frac{d^3\sigma(S_T)}{d^3\mathbf{P}_h} - E_h \frac{d^3\sigma(-S_T)}{d^3\mathbf{P}_h} = -2|S_T| \sum_{abc} \int dx_a \int dx_b \int d^2\mathbf{\kappa}_T \frac{1}{\pi z} \times \Delta_T f_a(x_a) f_b(x_b) \Delta_{TT} \hat{\sigma}(x_a, x_b, \mathbf{\kappa}_T) \Delta_T^0 D_{h/c}(z, \mathbf{\kappa}_T^2), \quad (16)$$

where, $\Delta_{TT} \hat{\sigma}$ is a partonic spin-transfer asymmetry.

The Sivers effect relies on T -odd distribution functions and predicts a single-spin asymmetry of the form

$$E_h \frac{d^3\sigma(S_T)}{d^3\mathbf{P}_h} - E_h \frac{d^3\sigma(-S_T)}{d^3\mathbf{P}_h} = |S_T| \sum_{abc} \int dx_a \int dx_b \int d^2\mathbf{k}_T \frac{1}{\pi z} \times \Delta_0^T f_a(x_a, \mathbf{k}_T^2) f_b(x_b) \frac{d\hat{\sigma}(x_a, x_b, \mathbf{k}_T)}{d\hat{t}} D_{h/c}(z), \quad (17)$$

where $\Delta_0^T f$ (related to f_{1T}^\perp) is a T -odd distribution.

Finally, the effect studied in [35] gives rise to an asymmetry

involving the other T -odd distribution, $\Delta_T^0 f$ (related to h_1^\perp):

$$\begin{aligned} E_h \frac{d^3\sigma(S_T)}{d^3P_h} - E_h \frac{d^3\sigma(-S_T)}{d^3P_h} \\ = -2|S_T| \sum_{abc} \int dx_a \int dx_b \int d^2k_T' \frac{1}{\pi z} \\ \times \Delta_T f_a(x_a) \Delta_T^0 f_b(x_b, k_T'^2) \Delta_{TT} \hat{\sigma}'(x_a, x_b, k_T') D_{h/c}(z), \end{aligned} \quad (18)$$

where $\Delta_{TT} \hat{\sigma}'$ is the partonic initial-state spin-correlation asymmetry.

3.2 Higher-twist and SSA's

As already mentioned, it was first pointed out in [22] that non-vanishing SSA's can also be generated in pQCD by resorting to higher twist and the so-called gluonic poles present in diagrams involving qqg correlators. Such asymmetries were later evaluated in the context of QCD factorisation in [23], where direct photon production was studied and, more recently, hadron production [36]. This program has now been further extended to cover the chirally-odd contributions in [37].

The possibilities multiply when higher-twist is taken into consideration as the new contribution can reside in any one of the three building blocks: parton densities, hard-scattering processes or fragmentation functions. Thus one has the following general expression:

$$\begin{aligned} d\sigma = \sum_{abc} \{ & G_F^a(x_a, y_a) \otimes f_b(x_b) \otimes d\hat{\sigma} \otimes D_{h/c}(z) \\ & + \Delta_T f_a(x_a) \otimes E_F^b(x_b, y_b) \otimes d\hat{\sigma}' \otimes D_{h/c}(z) \\ & + \Delta_T f_a(x_a) \otimes f_b(x_b) \otimes d\hat{\sigma}'' \otimes D_{h/c}^{(3)}(z) \}. \end{aligned} \quad (19)$$

The first term does not contain transversity and is the chirally-even mechanism studied in [36]; the second is the chirally-odd contribution analysed in [37]; and the third contains a twist-three fragmentation function $D_{h/c}^{(3)}$.

3.3 Phenomenology

Anselmino *et al.* [38] have compared the data to various models for partonic densities based on the previous possible (k_T) contributions and find good descriptions. However, they cannot yet differentiate between contributions. The higher-twist calculations of Qiu and Sterman [23] are rather opaque, involving many diagrams, complicated momentum flow, colour and spin structure. The twist-three correlators (as found in g_2) obey constraining relations with k_T -dependent densities; thus, in fact, the two approaches are not entirely independent. Indeed, it is well known that [39] that higher-twist may always be traded off for non-zero k_T .

4 A Novel Factorisation

The manner in which twist-three diagrams involving three-parton correlators (such as in Fig. 9) can supply an imagi-

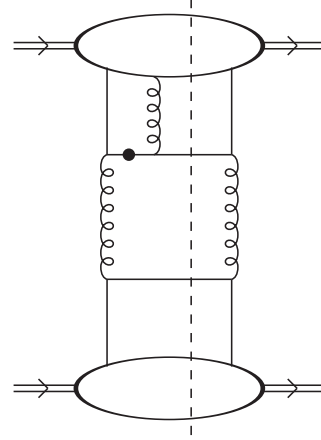


Figure 9. A typical twist-three diagram giving rise to pole terms.

nary part via a pole term (spin-flip is implicit in the particular operators considered, those relating to g_2 in DIS) is as follows [22]: the standard propagator prescription,

$$\frac{1}{k^2 \pm i\epsilon} = \mathbf{P} \frac{1}{k^2} \mp i\pi\delta(k^2), \quad (20)$$

then leads to an imaginary contribution for $k^2 \rightarrow 0$. Thus, mechanisms generating unsuppressed SSA's may easily be constructed. This is precisely the origin of the contributions successfully exploited in [23] to obtain large SSA's.

4.1 Pole Diagrams

The peculiar kinematics of the configuration involved may be further exploited to factorise the amplitude in a rather convenient and suggestive manner [40]. For a gluon $x_g p$ inserted into an (initial or final) external line p' , $k = p' - x_g p$ and this means $x_g \rightarrow 0$. The result is represented graphically in Fig. 10. The term $p' \cdot \xi$ contains the typical triple

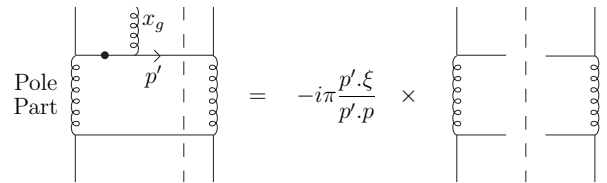


Figure 10. A graphical representation of the factorisation of the pole term.

product involving the spin vector shown in Eq. 1. The obvious interpretation is then that three-parton amplitudes in

general may be factorised into corresponding two-parton amplitudes multiplied by simple kinematical factors, in which all the spin information resides. Such factorisation can be performed systematically for all poles (gluon and fermion), *i.e.*, on all external legs with all possible soft insertions. This still generates rather complex structures: there are many possible such insertions for any given correlator, with contributions of different signs and momentum dependence.

4.2 Large- N_c

The structure of these twist-three contributions may be still further simplified by considering the colour structure of the various diagrams involved, which is also very different. In all cases (examined) it turns out that just one diagram, shown in Fig. 11, dominates in the large- N_c limit. All other

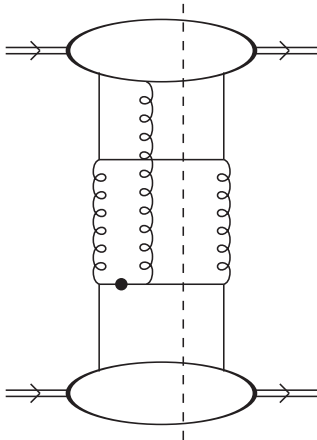


Figure 11. The single pole-term diagram surviving in the large- N_c limit.

possible insertions (leading to an imaginary part via a pole) are suppressed by $1/N_c^2$. Note that this is true irrespective of whether to final-state partons are quarks or gluons. At this point it becomes rather trivial to estimate (with the approximations made) the size and sign of SSA's that may be generated via such mechanisms in any given hadronic process.

We should note in concluding this section that the whole procedure still needs to be repeated for all the other twist-three contributions (*e.g.*, also in fragmentation). However, the simplifications demonstrated obviously render all such mechanisms very transparent, allowing easy and rapid evaluation of their impact in SSA.

Finally, for lack of time, I have not commented on and can only mention other recent developments concerning, for example, non-standard time reversal [41], final-state interactions [42] and non-trivial Wilson lines [43].

5 Conclusions and Outlook

Single-spin asymmetries have passed from a dark age, during which there was essentially *no* (QCD-based) theory, to a period of illumination, where there is almost *too much*. Hopefully, the multiplicity of contributions can be reduced to a few simple terms:

- experiment can eliminate some possibilities if null results are obtained;
- relationships between three-parton correlators and k_T -dependent densities should show the equivalence between phenomenological models;
- pole-factorisation and the large- N_c limit should simplify calculations and allow a simple pattern to emerge.

Fortunately, the experimental activity, now on the increase, is matched by a continued interest in the phenomenology on the part of a long-standing group of spin theorists. This is thus certainly an area of hadronic physics that is destined to produce interesting (and perhaps) surprising results in the not too distant future.

Somewhat in contrast, the case of transversity is rather cut-and-dried as far as the theoretical interpretation is concerned. What is now lacking is experimental knowledge of this spin density. Such data will complete our understanding of the spin structure of the proton. Moreover, the peculiar nature of the pQCD evolution of transversity (it is of the pure non-singlet form) could, in principle, allow interesting studies of scaling violations.

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